HEAT TRANSFER FOR MELTING ABOUT A HORIZONTAL CYLINDER

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ABSTRACT

Heat transfer in a latent thermal energy storage during melting process heated by a surface of constant wall temperature has been studied experimentally and analytically. Theoretically, the melt layer for the subcooled PCM is thinner than that for an-unsubcooled. The experimental results show that the reaching time of melting temperature for the points located at the same radius depend on their angular positions. The point located or the vertical axis arrive faster at the melting temperature than those at 45° from that axis. From time-temperature histories point of view, the experimental results show the reaching time of melting temperature at the particular points are faster than the theoretical ones. The two facts mentioned above indicate that during melting process, natural convection occurs in the liquid phase and supports the interface at the upper part of the tube.

INTRODUCTION

Heat conduction problems involving solid-liquid phase change are of interest in a wide range of technologies and geophysics. Most of analytical solutions from Bell,G.E., 1979, Carslaw, H.S., et al., 1959 and Katayama, K., et al., 1981 are concerned with transients in which, initially the PCM (Phase Change Medium) at its melting temperature. It also appears in various heat conduction textbooks that the effect of different densities between the solid and liquid phase receives a little attention. The aim of this study is to carry out the heat transfer theoretically and experimentally for melting about horizontal cylinder imbedded in PCM. The PCM used in this study is paraffin-wax. The heating process is at constant wall temperature.

EXPERIMENTAL APPARATUS AND PROCEDURE

The experimental apparatus used in this work is the shell-tube type heat exchanger as shown in Figure 1. The PCM fills the space between tube and shell. The diameter of copper tube and plexiglas shell are 1 and 3 inch respectively. The physical properties of the PCM, Himran, S., et al., 1994, are as follow:

- Melting temperature: \( T_m = 46.7°C \)
- Thermal conductivity: \( k_s = k_c = 0.1383 \text{ W/m}^\circ\text{C} \)
- Specific heat: \( c_s = c_c = 2850 \text{ J/kg}^\circ\text{C} \)
- Density: \( \rho_s = 947 \text{ kg/m}^3 \); \( \rho_c = 750 \text{ kg/m}^3 \)
- Coefficient of volumetric thermal expansion: \( \beta = 0.0021 \text{ K}^{-1} \)
- Kinematic viscosity: \( \nu = 2.5 \times 10^{-6} \text{ m}^2/\text{s} \)
- Latent heat of fusion: \( L = 20000 \text{ J/kg} \)

![Figure 1. Experimental apparatus](image)
ANALYSIS

Since the flow rate of the working fluid is so high, that the analysis could be performed as radially one-dimensional. The on-dimensional transient heat conduction problem with melting is formulated in a cylindrical co-ordinate system where modeled as shown in Figure 3. The basic equations, the boundary and the initial conditions are:

Initial conditions:  

\[ T_i = T_0, \quad t < 0 \]

It is convenient to non-dimensionalize the above set of equations by introducing the variables:

\[
\theta_j = \frac{T_j - T_m}{T_e - T_m}, \quad j = \text{a.s.i.n.m.}
\]

\[ R = \frac{r}{\theta_j} (S = \frac{\pi}{2}; \theta_0 = \frac{\pi}{2}; \theta_i = \frac{\pi}{2}), \quad \tau = \frac{t}{T_e} = 0.7920 \]

The above equations become:

- **Liquid phase**:

\[
\frac{d^2 \theta_l}{dr^2} + \frac{1}{r} \frac{d \theta_l}{dr} = \frac{d \theta_i}{dr}, \quad 0 < R < S, \quad t > 0
\]

(1)

- **Solid phase**:

\[
\frac{d^2 \theta_s}{dr^2} + \frac{1}{r} \frac{d \theta_s}{dr} = \frac{d \theta_i}{dr}, \quad S < R < \infty, \quad t > 0
\]

(2)

- **Boundary conditions**:

\[ \theta_i = 1, \quad R = 1, \quad t > 0 \]

\[ \theta_i = 0, \quad R = \infty, \quad t > 0 \]

(3)

Liquidsolid interface: \( \tau = 0, \quad R = 0 \)

continuity of temperature: \( \theta_i = \theta_s \)

(4a)

Energy balance: \( \frac{d^2 \theta}{dr^2} = \frac{L}{\rho C_p} \frac{d \theta}{dr} \)

(5)

initial condition: \( \theta_i = 0, \quad t < 0 \)

To solve the partial differential equation (1) or (2), we introduce the transformation parameter:

\[ \tau = \frac{r}{a} \]

Employing the chain rule for:

\[
\frac{d^2 \theta}{dv^2} = \frac{d \theta}{dz} \left( \frac{d^2 v}{dz^2} + \frac{d \theta}{dz} \frac{dv}{dz} \right)
\]

and utilizing the transformation parameter, we obtain the homogeneous linear ordinary differential equation of order 1 as follows:

\[
\frac{d^2 \theta}{dZ^2} + \frac{1}{Z} \frac{d \theta}{dZ} + \frac{\theta}{Z} = 0
\]

(6)

By using the method of reduction of order, Hildbrand, F.B., 1976, we find the primitive solutions for the solid and liquid phases which fulfill the above partial differential equations. These equations are as follows:

Liquidsolid phase:

\[ \theta_i = \text{A} + \text{B} \text{e}^{-\frac{Z}{d}} \]

(7)
Solid phase: \( \theta_i = C + D \left( \frac{R_i^2}{4 \eta} \right) \)

\( \left( \frac{R_i^2}{\tau} \right) \) is exponential integral function, and its properties could be consulted reference, Abramowitz, M., et al., 1964. The constant A, B, C, and D are determined by introducing the boundary conditions (3), (4b), (5), and we find the complete solutions for the temperature distributions in liquid and solid phases.

Liquid phase:

\[
\theta_i = \theta_0 - \left( 1 - \theta_0 \right) \frac{R_i^2}{4 \eta} \left[ \frac{1}{\eta} Ei \left( \frac{R_i^2}{4 \eta} \right) + \frac{1}{\eta} Ei \left( \frac{R_i^2}{4 \eta} \right) \right]
\]

Solid phase:

\[
\theta_i = \frac{R_i^2}{4 \eta} \left( \frac{1}{\eta} Ei \left( \frac{R_i^2}{4 \eta} \right) \right)
\]

To estimate the heat flux during melting can be easily obtained from the equation: 

\[
q = -k \frac{\partial T}{\partial r} \bigg|_{r=r_i}
\]

By introducing equations (9) we obtain the heat flux:

\[
q = -k \frac{2}{R} \left( \frac{1}{\eta} Ei \left( \frac{R_i^2}{4 \eta} \right) \right)
\]

The parameters \( \lambda_i^2 = \frac{S_i^2}{4 \eta} \), \( \lambda_s^2 = \frac{S_s^2}{4 \eta} \) should be satisfied for all times, so that they must be a constant. When the equations (9) and (10) are substituted into equation (4b), we obtain the following transcendental equation, where \( \lambda \) is the root of the equation:

\[
\frac{(1 - \theta_0)}{\eta} - \frac{\theta_0 \lambda^2}{\eta} + \frac{1}{\eta} Ei \left( \frac{R_i^2}{4 \eta} \right) = 0
\]

If we introduce the above physical properties of the PCM into equations (12) we find:

\[
\frac{0.2556}{\eta} - \frac{0.7444 \lambda^2}{\eta} + \frac{2.5321}{\eta} Ei \left( \frac{R_i^2}{4 \eta} \right) = 0
\]

where: \( \lambda^2 = 1.2627 \lambda_i^2 \)

The position of the liquid-solid interface is given by \( S = 2 \lambda_i \sqrt{\tau} \)

RESULTS AND DISCUSSIONS

The time-temperature histories for four points of measurements imbedded in PCM (points 1, 2, 3, 4) are recorded during melting process. The period of measurement is 165 minutes, where the interface (melting front) has been passed the four thermocouples.

The root \( \lambda_i \) could be obtained by solving numerically the transcendental equation (13) and using bisection method (James, M.L., et al., 1985). The calculations are performed by digital computer using Turbo Pascal program. Introducing the value of \( \tau_1 \) into the calculation program we find \( \tau_2 \), and the interface position \( S \) as function of \( \tau_1 \) (interface movement), we can obtain the variation of the temperature in liquid and solid phase (time-temperature histories) and the variation of heat flux, i.e. by introducing \( S(\tau) \) into the equations (9), (10) and (11) respectively. To get the time-temperature histories at \( \tau_1 = 14.7 \text{ min} \) and \( \tau_2 = 17.7 \text{ min} \), we substitute \( R = R_1, \tau_2 = 1.1575 \) and \( R = R_2, \tau_2 = 1.3937 \) into equations (9) and (10) respectively.

Figure 4 shows the variations of interface for subcooled and unsubcooled PCM during melting process. In this case, the subcooled and unsubcooled PCM mean that the PCM at the atmospheric and at 5°C below melting temperature respectively. The subcooled and unsubcooled curve could be obtained by putting the initial condition of PCM \( T_1 = 25.44 \text{ °C} \) and 41.7°C (5°C below the melting temperature) respectively. The interface of unsubcooled curve is higher than subcooled one. This shows that the presence of subcooled, the melt layer is thinner than it would be without subcooled.

![Figure 4. Variation of interface for Dubcooled and Unsubcooled PCM](image-url)
In Figure 5, the points of measurement: 1, 2, 3, and 4 locate at higher positions than the points on the subcooled curve at the particular time. This discrepancy is due to the natural convection in the liquid phase. The natural convection will cause melting to proceed more rapidly than would be expected when heat transfer is conduction alone. During the early of melting, the melt layer is very thin and the heat transfer is conduction. The melt layer is symmetrical about the axis of the cylinder. The layer thickness $\delta$ is predicted by the criteria that for the pseudo-conductive regime $Ra = 1700$ (Grigull, U., et al., 1968), and we obtain $\delta = 1.22$ mm ($=13.92$ mm) with the heating time $54.05$ minutes.

Figure 5. Variation of interface for subcooled and experimental of the point 1, 2, 3, 4.

Experimental results show that the reaching time of melting temperature for the points located at the same radius depend on the angular position from the vertical axis of the tube cross section (see Figure 2). The reaching times of melting temperature for the points 1 and 2 are 63 and 66 minutes and for the points 3 and 4 are 129 and 150 minutes respectively. These indicate that after some time ($t > 54.04$ minutes) the natural convection develops and influences the melt shape around the surface of the tube. According to Bejan, A., 1993, the plume has been originated from the top of the tube and contributed to the melt layer. This natural convection supports the interface at the upper side of the tube.

Figures 6a and 6b show the actual and theoretical time-temperature histories at the point of measurement 1, 2, and 3, respectively. The disagreements between the data are due to natural convection occurred in the melting layer.

Figure 6a. Time temperature history at the radius $r = 14.7$ mm

Figure 6b. Time temperature history at the radius $r = 17.7$ mm

Figure 7 shows the heat flux variation during melting process. The unsupplied curve is lower that the subcooled one which indicates that part of heat input is used to increase the temperature of the solid to its melting temperature.

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CONCLUSIONS

The analytical and experimental results concerning the melting process from a horizontal tube imbedded in paraffin-wax lead to the following conclusions.

1. The analytical results show that the variation of the subcooled interface is lower than the unsubcooled and the variation of heat flux of subcooled is higher than the unsubcooled. These facts indicate that part of the heat input is absorbed as sensible heat by the solid to increase its temperature. Furthermore, the melt layer of subcooled is thinner than it would be of unsubcooled.

2. The experimental results of temperature histories deviate from the analytical result. It is understood that the natural convection has been occurred during melting process, although heat conduction is dominant in the very early stage of heating. The natural convection will cause melting to proceed more rapidly than would be expected when heat transfer is pure conduction. The natural convection will cause the plume which has originated from the top of the tube and converged to an asymmetrical melt layer around the surface of the tube. The plume conveys hot liquid to the upper part of the melt region and continues support the upward movement of the interface.

REFERENCE