Welfare Measurement: Compensation Variation And Equivalent Variation Approach

The deep economic crisis in Indonesia has had implications for poor and rich household's welfare in both rural and urban areas. An Asian Development Bank Report (ADB 1999) stated that the poor have been most hit by the economic crisis. Indonesia had 21.9 million poor people in 1995 and a 25 percent increase in the poverty line results in more than doubling the head count index, from 11 to 25 in 1996 (ADB 1999: 77). In Indonesia poor people are most likely to stay in rural areas, therefore it might be believed that the society most affected by the economic crisis is rural society.

The individual household has decreases in welfare due to the economic crisis through two effects i.e. decreasing in household's income and increasing of commodity prices. The decrease in income is not only because of decrease in wage rates but also, more fundamentally, because the fall in the demand for labour leads to unemployment. A lot of labour intensive industries -i.e. real estate and property, services, construction, banking etc- are hit by the economic crisis, therefore they have to reduce the amount of labour employed. In consequence, unemployment is a social phenomenon that might not be avoided.

The second effect is the increase of commodity prices that automatically lower the purchasing power of households. In other words, the household's real income decreases. Again, the poor people are deeply affected by the economic crisis. But, it is widely believed that the decrease of labour demand and the increase of commodity prices are more serious in urban than rural areas (ADB 1999:80). Therefore, it was believe that the society mostly affected by the economic crisis.
is urban society. One of the conclusions of the recent survey conducted by Sumarto, Wetterberg and Pritchett (1999) by interviewing three expert respondents in every kecamatan (sub-district) in Indonesia is that urban areas have been harder hit by the economic crisis than rural areas. Evidence on whether rural or urban households have borne the brunt is ambiguous. There might be little argument that both households have been affected, although not necessarily for the same reason. On the one side, the relative increase in the price of agriculture products has provided net producers some protection from crisis. On the other hand, the agricultural sector has been the primary absorbing sector for employment (Frankenberg, Thomas and Beegle 1999). The ADB Report (1999:80) describes that rural areas are also affected:

This is not essentially an urban shock, despite the high profile of urban unemployment figures. Rural areas will also seriously affected by labor movements, production linkages, and intra-household relationship because of the highly integrated nature of the urban and rural economies and the declining in urban areas. Increased underemployment and falling wages may be more widespread and valid indicators of a decline in well being than unemployment statistics.

The welfare measurement is therefore extremely important. This paper is addressed to derive the welfare change measurement that probably could be applied in the empirical study, namely Equivalent Variation (EV) and Compensation Variation (CV). The rest of this paper is organized as follows. Part 2 explains EV and CV. In Part 3, the linear expenditure system (LES) is exposed and then it is followed by derivation EV and CV in the LES framework in part 4. And some conclusions are withdrawn in part 5.

**Equivalent Variation and Compensation Variation (CV)**

The economic crisis has brought some increases in food prices and decreases of the household's income. The Equivalent Variation (EV) and Compensation Variation (CV) will be applied to analyze the impact of the economic crisis on economic welfare. The Equivalent Variation (EV) can be defined as the dollar amount that the household would be indifferent to in accepting the changes in food prices and income (wealth). It is the change in her/his wealth that would be equivalent to the prices and income changes in terms of its welfare impact (EV is negative if the prices and income changes would make the household worse off).

Meanwhile, the Compensating Variation (CV) measures the net revenue of the planner who must compensate the household for the food prices and income changes, bringing the household back to its welfare (utility level) (Mas-Colell, A., Whinston, M.D. and Green, J.R., 1995:82). The CV is negative if the planner would have to pay household a positive level of compensation because the prices and income changes make household worse off) Figure 1 visualizes the EV and CV when there is only an increase in price of one good.

If there are changes in prices and income, the EV and CV can be formulated as:

\[ EV = E(p' \cdot U) - E(p \cdot U) + (M' - M) \] .......(1)

\[ CV = E(p' \cdot U) - E(p \cdot U) + (M - M') \] .......(2)

where:
- E is expenditure function
- \( p' \) is an original price vector
- \( p' \) is a new price vector
- \( M' \) is original income level
- \( M' \) is new income level

**Linear Expenditure System**

Technically, the welfare change could be measured by how much money is needed by households to compensate the change in food prices and income originating in Hicks (1939), namely Equivalent Variation (EV) and Compensating Variation (CV). The Equivalent Variation (EV) can be seen as the dollar amount that the household would be indifferent to in accepting the changes in food prices and income (wealth). The Compensating Variation (CV) measures the net revenue of the planner who must compensate the household for the food prices and income changes, bringing the household back to its welfare (utility level) (Mas-Colell, Whinston and Green 1995:82).

**Estimate Demand, Indirect Utility and Expenditure Function**

To measure the welfare change, we have to estimate the household expenditure function. To do that some steps should be followed. Firstly, the household utility function should be established. And in this study, the household's utility function is assumed to be Cobb-Douglas which can derive the Linear Expenditure System of demand (Stone, 1954). This assumption is taken because the Linear Expenditure System (LES) is suitable for the household food consumption/demand. Secondly, the Linear Expenditure System of
household demand can be estimated by using available data. Therefore the household demand function (Marshallian and Hicksian) for each food commodity can be found.

From the estimated demand function, we can derive the household indirect utility and expenditure function. Finally, the welfare change can be measured by comparing the household expenditure pre-crisis and post-crisis to get the same utility (welfare). These stages will be expressed in the next paragraphs.

**Marshallian Demand System**

In this study, it is assumed that the rural and urban households have a utility function following the more general Cobb-Douglas. Stone (1954) made the first attempt to estimate a system equation explicitly incorporating the budget constraint, namely the Linear Expenditure System (LES). In the cases of developing countries, this system has been used widely in the empirical studies in India by some authors (Pushparaj and Asok (1964), Bhattacharya (1967), Joseph (1968), Raina (1985), Satish and Sanjib (1999)).

Formally the individual household's preferences defined on n goods are characterized by a utility function of the Cobb-Douglas form. Klein and Rubin (1948) formulated the LES as the most general linear formulation in prices and income satisfying the budget constraint, homogeneity and Slutsky symmetry. Basically, Samuelson (1948) and Geary (1950), derived that the LES representing the utility function:

\[ U(X_1, X_2, \ldots, X_n) = (X_1 - X_1^*)^\alpha_1 (X_2 - X_2^*)^\alpha_2 \cdots (X_n - X_n^*)^\alpha_n \]

In brief, it can be expressed as:

\[ U(X_i) = \prod_{i=1}^{n} (X_i - X_i^*)^{-\alpha_i} \]

The individual household's problem is to choose \( x_i \) that can maximize its utility \( U(x_i) \) subject to its budget constraint. Therefore, the optimal choice of \( x_i \) is obtained as a solution to the constrained optimization problem as follows:

\[
\text{Max } U(X_i) = \prod_{i=1}^{n} (X_i - X_i^*)^{-\alpha_i} \\
\text{Subject to: } PX \leq M
\]

To solve the problem, the Lagrange method can be applied. The Lagrange formula for this problem is:

\[
\text{Max } \Omega = U(X_i) = \prod_{i=1}^{n} (X_i - X_i^*)^{-\lambda} + \lambda (M - PX), \ldots \ldots (5)
\]

\[
\begin{bmatrix}
\frac{\partial \Omega}{\partial x_1} = 0 \\
\frac{\partial \Omega}{\partial x_2} = 0 \\
\vdots \\
\frac{\partial \Omega}{\partial x_n} = 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
\vdots \\
x_n
\end{bmatrix}
\begin{bmatrix}
\sum_{i=1}^{n} \alpha_i \\
\alpha_2 \\
\vdots \\
\alpha_n
\end{bmatrix}
\begin{bmatrix}
\lambda \\
\lambda \\
\vdots \\
\lambda
\end{bmatrix}
\begin{bmatrix}
P_1 \\
P_2 \\
\vdots \\
P_n
\end{bmatrix}
\begin{bmatrix}
M \\
0 \\
\vdots \\
0
\end{bmatrix}
\]

Where: \( \lambda \) is the Lagrange multiplier. It is interpreted as the marginal utility of income showing how much the individual household's utility will increase if the individual household's income \( M \) is increased by \$1.

Take the derivatives and get the first order conditions (FOCs):

\[
\frac{\partial \Omega}{\partial x_i} = \alpha_i (X_i - X_i^*)^{-\lambda} P_i = 0 \quad \ldots \ldots (6)
\]

\[
\text{where: } U(X_i) = \prod_{i=1}^{n} (X_i - X_i^*)^{-\alpha_i}
\]

\[
\frac{\partial \Omega}{\partial \alpha_i} = M - PX = 0 \quad \ldots \ldots (7)
\]

In matrix form (6) and (7) can be represented as follows:

Equation (4) tells us that the marginal utility of \( x_i \) is equal with the marginal utility of income multiplied by price of \( x_i \). From (6) and (7), we have \( n \times n \) unknown variables \( (x_1, x_2, \ldots, x_n, \alpha_1, \ldots, \alpha_n) \) and \( n+1 \) equations. By applying Cramer's rule, the unknown variables \( (x_1, x_2, \ldots, x_n) \) can be found:

\[
\begin{bmatrix}
x_1 \\
x_2 \\
\vdots \\
x_n
\end{bmatrix} = \frac{1}{|A|} \begin{bmatrix}
1 \\
1 \\
\vdots \\
1
\end{bmatrix}
\]

Where \(|A|\) is the determinant of matrix \( A \), which is constructed from matrix \(|A|\) by replacing the first column of \( A \) with matrix \( C \). And the \( A \) is the determinant of matrix \( A \). The other demands \( (x_1, x_2, \ldots, x_n) \) and \( \lambda \) can be found by applying equation (8) in the same way. From (8), we can find the Marshallian (uncompensated) demand function for commodity \( x \), as follows:

\[
x_i = x_i^* \frac{\alpha_i (M - \sum_{j=1}^{n} P_j x_j)}{P_i} \quad \text{for all } i \text{ and } j \ldots \ldots (9)
\]

\[
\text{Where: } i \in (1, 2, \ldots, n) \quad j \in (1, 2, \ldots, n)
\]

Since a restriction that the sum of parameters \( \alpha_i \) equals to one, \( \sum_{i=1}^{n} \alpha_i = 1 \), is imposed equation (9) becomes:

\[
x_i = x_i^* \frac{\alpha_i (M - \sum_{j=1}^{n} P_j x_j)}{P_i} \quad \text{for all } i \text{ and } j \ldots \ldots (10)
\]
Equation (9) can be also reflected as the Linear Expenditure System as follows:

\[ p \times x_i = p_i \times x_{i}^{*} + \alpha_i \left( M - \sum_{j=1}^{n} p_j \times x_j^{*} \right) \]

for all i and j ... (11)

This equation system (11) can be interpreted as stating that expenditure on good i, given as \( p \times x_i \), can be broken down into two components. The first part is the expenditure on a certain base amount \( x_{i}^{*} \) of good i, which is the minimum expenditure to which the consumer is committed (substitution expenditure), \( p_i x_{i}^{*} \) (Stone 1954). Samuelson (1948) interpreted \( x_{i}^{*} \) as a necessary set of goods resulting in an informal convention of viewing \( x_{i}^{*} \) as non-negative quantity. The restriction of \( x_{i}^{*} \) to be non-negative values however is unnecessarily strict. The utility function is still defined whenever \( x_{i} - x_{i}^{*} > 0 \). Thus the interpretation of \( x_{i}^{*} \) as a necessary level of consumption is misleading (Pollak, 1968). The \( x_{i}^{*} \) allowed to be negative provides additional flexibility in allowing price-elastic goods. The usefulness of this generality in price elasticity depends on the level of aggregation at which the system is treated. The broader the category of goods, the more probable it is that the category would be price elastic. Solarli (in Howe 1954:13) interprets negativity of \( x_{i}^{*} \) as superior or delux commodity.

In order to preserve the committed quantity interpretation of the \( x_{i}^{*} \)s when some \( x_{i}^{*} \) are negative, Solarli (1971) redefines the quantity \( \sum_{j=1}^{n} p_j \times x_j^{*} \) as 'augmented supernumerary income' (in contrast to the usual interpretation as supernumerary income, regardless of the signs of the \( x_j^{*} \)). Then, defining \( n^{*} \) such that all goods with \( n^{*} \) have positive \( x_{i}^{*} \) and goods for \( n^{*} \) are superior with negative \( x_{i}^{*} \), Solarli interprets \( \sum_{j=1}^{n} p_j \times x_j^{*} \) as supernumerary income and \( \sum_{j=1}^{n} p_j \times x_j^{*} \) as fictitious income. The sum of 'Salary-supernumerary income' and fictitious income equals augmented supernumerary income. Although somewhat convoluted, these definitions allow interpretation of 'Supernumerary income' as expenditure in excess of the necessary to cover committed quantities.

The second part is a fraction \( \alpha_i \) of the supernumerary income, defined as the income above the 'subsistence income' \( \sum_{j=1}^{n} p_j \times x_j^{*} \) needed to purchase a base amount of all goods. The \( \alpha_i \) are scaled to sum to one to satisfy the budget demands. The \( \alpha_i \) is referred to as the marginal budget share, \( \alpha_i / \alpha \). It indicates the proportion in which the incremental income is allocated.

As stated above, the Linear Expenditure System (LES) satisfies the conditions of:
(i) homogeneity of degree zero in prices
(ii) the budget constraint (Engel Aggregation and Cournot Aggregation conditions)

(iii) Slutsky conditions (negative and symmetry conditions)

by construction. In combination with fourth i.e. the negative semi-definiteness of the Slutsky-Hicks substitution term matrix, they ensure that the demand function in question is generated by the maximization of utility function. Those conditions lead to some restrictions. First, the \( \alpha_i \)s are positive which is incorporated in the specification of the utility function. Second, the sum of the marginal budget share is equal to one so \( \sum_{i=1}^{n} \alpha_i = 1 \) that results in demand system of the form shown in equation (11). Third, inferior and complementary goods are not allowed. However, at the high level of aggregation employed in this study, this limitation (inferior and complementary) is not very restrictive. The higher the level of aggregation, the less likely it is that consumption of any given category would decline with the increase in income and some \( \alpha_i \)s could be negative (Howe 1974:18).

The LES is widely used for three reasons. First, it has a straightforward and reasonable interpretation. Second, it satisfies the theory of demand (theoretical restrictions). Third, it can be derived from a specific utility function (the Stone-Geary or Klein-Rubin utility function) (Intriligator, Bodkin and Haldal 1996:255).

**Indirect Utility and Expenditure Function**

The indirect utility function \( U(P,M) \) can be found by substituting the new Marshallian demand \( x_i \) (equation 7b) into the utility function \( U(x) \) (equation 1). Therefore the indirect utility function is:

\[ U(P,M) = \prod_{i=1}^{n} \left( \frac{\alpha_i}{p_i} \right)^{\alpha_i} \left( \sum_{i=1}^{n} p_i x_i^{*} \right)^{1-\alpha_i} \]

for all i and j (12)

**Hicksian Demand**

By derivation the expenditure function \( E(P,U) \) with respect to a particular price (using the Shephard lemma), the Hicksian demand function can be represented as:

\[ h_i = \frac{\partial E}{\partial p_i} = x_i^{*} \frac{\alpha_i}{\alpha} (1 - \prod_{i=1}^{n} \left( \frac{p_i}{p_i^0} \right)^{\alpha_i}) \left( \sum_{i=1}^{n} p_i x_i^{*} \right)^{-\alpha} \]

for all i (13)

**4. CV and EV in the LES Framework**

In the context of Linear Expenditure System (LES), equation (1) and (2) become:

\[ E V = \left( \prod_{i=1}^{n} \left( \frac{P_i}{P_i^0} \right)^{\alpha_i} \right)^{-1} M^* - \prod_{i=1}^{n} \left( \frac{P_i}{P_i^0} \right)^{\alpha_i} \sum_{i=1}^{n} p_i x_i^{*} \left( M^* - M^* \right) \]

for all i and j

Where:
- \( P^* \) is food commodity prices pre-economic crisis vector
- \( P^* \) is food commodity prices post-economic crisis vector (during economic crisis vector)
- \( P^* \) is food commodity prices pre-economic crisis vector
- \( P^* \) is food commodity prices post-economic crisis vector (during economic crisis vector)
- \( U^0 \) is level of utility (welfare) pre-economic crisis
- \( U^0 \) is level of utility (welfare) post-economic crisis
- \( M^* \) is income (expenditure) pre-economic crisis vector
- \( M^* \) is income (expenditure) post-economic crisis vector
- \( M^* \) is income (expenditure) pre-economic crisis vector
- \( M^* \) is income (expenditure) post-economic crisis vector
By knowing the change in prices and income due to the economic crisis, we can find the change in welfare measured by CV and EV. The EV and CV indicate whether the household is worse off or better off under the economic crisis. This will answer the first question of this research i.e. how much the individual household should be compensated due to the economic crisis to hold the same utility (welfare). And by comparing the welfare change of the urban and rural individual households, we can answer the second question of which society, rural or urban, is most effected by the economic crisis.

5. Conclusion
Welfare decreases when prices increase or income decreases. The impact of income decrease on welfare (EV and CV) is virtually clear. It only deals with shifting of budget line. But, the impact of price changes is rather complicated because it relates with the rotation of budget line, moreover, when we deal with the increase in some prices and the decrease in the others.

References

The difference between the impossible and the possible lies in our own imagination, which is the beginning of our success or failure.